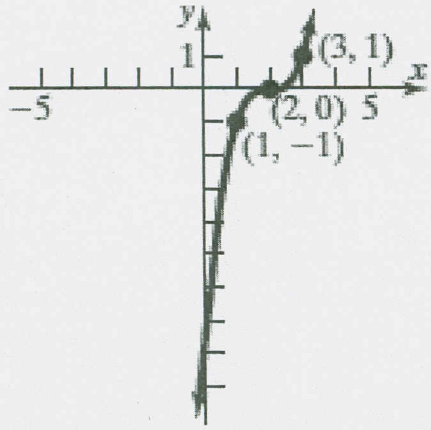
#s 23 – 36: Use transformations of the graph of $y = x^4$ or $y = x^5$ to graph each function.

24. $f(x) = (x-2)^5$

24.



#s 37 – 44: Form a polynomial whose zeros and degree are given.

42. Zeros: −3, −1, 2, 5; degree 4

$f(x) = a(x+3)(x+1)(x-2)(x-5)$ for any constant a is main idea. You should be able to (and it's good practice) expand and get something like the following, if you choose a to be 1:

42. $f(x) = x^4 - 3x^3 - 15x^2 + 19x + 30$ for $a = 1$

43. Zeros: −1, multiplicity 1; 3, multiplicity 2; degree 3

Factored form: $(x+1)(x-3)^2$

Expanded form: $x^3 - 5x^2 + 3x + 9$

In Problems 65–88:

(a) List each real zero and its multiplicity.

(b) Determine whether the graph crosses or touches the x-axis at each x-intercept.

(c) Determine the behavior of the graph near each x-intercept (zero).

(d) Determine the maximum number of turning points on the graph.

(e) Determine the end behavior; that is, find the power function that the graph of f resembles for large values of $|x|$.

(f) Put all the information together to obtain the graph of f.

51. $f(x) = (x-5)^3(x+4)^2$

(a) $x=5,\ m=3$; $x=-4,\ m=2$

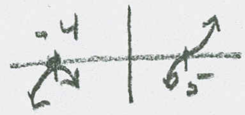
(b) $x=5$ cross ; $x=-4$, touch

(c) Near $x=5$: $(x-5)^3(5+4)^2 \approx 9(x-5)^3$
Looks like $+(x-5)^3$.
Near $x=-4$: $(-4-5)^3(x+4)^2 \approx -729(x+4)^2$
Looks like $-(x+4)^2$

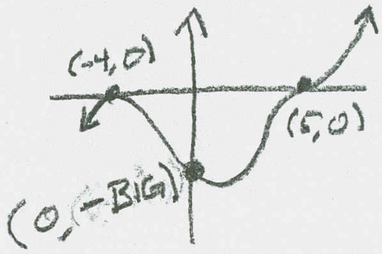
(d) Max # of turns: $n-1 = 5-1 = 4$

(e) $f(x) = (x-5)^3(x+4)^2 \xrightarrow{|x| \to \infty} (x)^3(x)^2 = x^5$

(f)



$f(0) = (-5)^3(4)^2 = -\text{BIG}!$



52. $f(x) = (x+\sqrt{3})^2(x-2)^4$

(a) $x = -\sqrt{3},\ m=2$; $x=2,\ m=4$

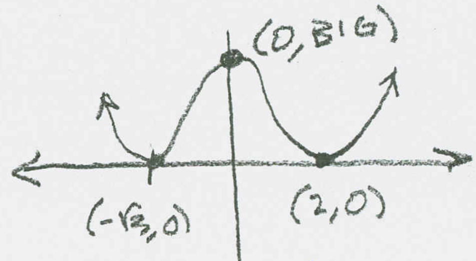
(b) $x=-\sqrt{3}$, touch; $x=2$, touch

(c) Near $x=-\sqrt{3}$:
$(x+\sqrt{3})^2(-\sqrt{3}-2)^4$ Looks like
$+(x+\sqrt{3})^2$
Near $x=2$: $(2+\sqrt{3})^2(x-2)^4$. Looks like $+(x-2)^4$

(d) Max # turns: $n-1 = 6-1 = 5$

(e) $f(x) \xrightarrow{|x| \to \infty} (x)^2(x)^4 = x^6$

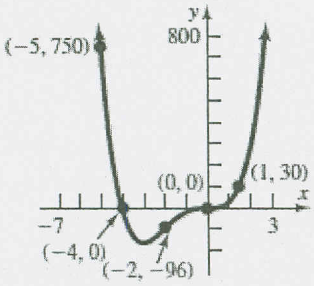
(f)



This graph has 3 turning points, we have no way of knowing if there are more.

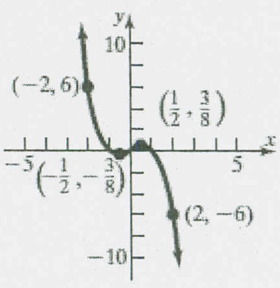
69. $f(x) = 6x^3(x+4)$

69. **(a)** x-intercepts: $-4, 0$; y-intercept: 0
(b) Crosses at $-4, 0$
(c) $y = 6x^4$
(d) 3
(e) Near -4: $f(x) \approx -384(x+4)$;
Near 0: $f(x) \approx 24x^3$.
(f)



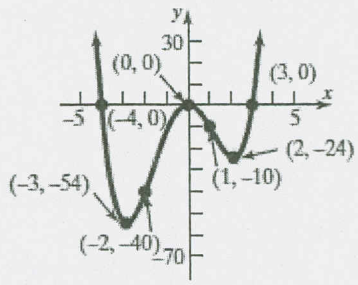
76. $f(x) = x - x^3$

76. $f(x) = x - x^3 = -x(x^2 - 1)$
$= -x(x-1)(x+1)$
(a) x-intercepts: $-1, 0, 1$; y-intercept: 0
(b) Crosses at $-1, 0$, and 1 **(c)** $y = -x^3$
(d) 2
(e) Near -1; $f(x) \approx -2(x+1)$; Near 0: $f(x) \approx x$; Near 1: $f(x) \approx -2(x-1)$
(f)



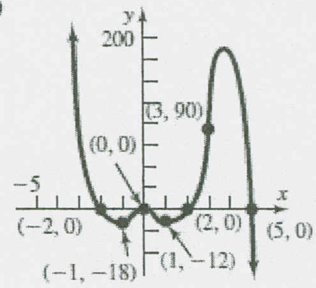
78. $f(x) = x^2(x-3)(x+4)$

78. **(a)** x-intercepts: $-4, 0, 3$; y-intercept: 0
(b) Crosses at -4 and 3; touches at 0
(c) $y = x^4$ **(d)** 3
(e) Near -4: $f(x) \approx -112(x+4)$;
Near 0: $f(x) \approx -12x^2$;
Near 3: $f(x) \approx 63(x-3)$
(f)



88. $f(x) = -x^2(x^2 - 4)(x - 5)$

88. **(a)** x-intercepts: $-2, 0, 2, 5$; y-intercept: 0
(b) Touches at 0; crosses at -2, 2, and 5
(c) $y = -x^5$ **(d)** 4
(e) Near -2: $f(x) \approx -112(x+2)$;
Near 0: $f(x) \approx -20x^2$;
Near 2: $f(x) \approx 48(x-2)$;
Near 5: $f(x) \approx -525(x-5)$
(f)



You might want to plug in other values, to assure yourself that you got it right, but we're pretty much expected to go with info taken from steps (a) — (e). And even step (e) is just insurance, since "touch" or "cross" from step (b) is enough to get the general idea of the graph. BUT the take-home test might include step (e) – type analysis.